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TECHNICAL REPORT H-77-18

A MATHEMATICAL MODEL FOR UNSTEADY-FLOW COMPUTATIONS THROUGH THE COMPLETE SPECTRUM OF FLOWS ON THE LOWER OHIO RIVER

by

Billy H. Johnson

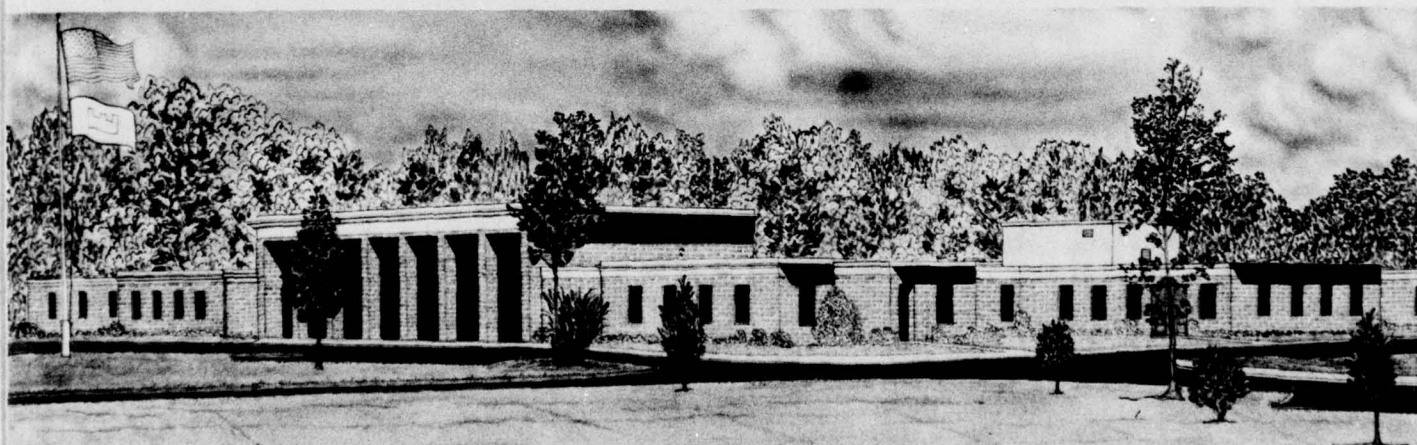
Hydraulics Laboratory

U. S. Army Engineer Waterways Experiment Station
P. O. Box 631, Vicksburg, Miss. 39180

October 1977

Final Report

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20. ABSTRACT (Continued).

at Cairo, Illinois. A mathematical model, SOCHMJ, for unsteady-flow solutions in multi-junction systems has been modified to include the effect of five navigation dams on the Ohio River in addition to the effect of operations at Barkley and Kentucky Reservoirs. For economical applications of the model to the system extending from Louisville, Kentucky, on the Ohio River, Livermore, Kentucky, on the Green River, Mt. Carmel, Indiana, on the Wabash River, Barkley Dam on the Cumberland River, Kentucky Dam on the Tennessee River, and Cape Girardeau on the upper Mississippi River to Caruthersville on the lower Mississippi River, the model has been modified to accept two time steps. A "large" time step applies to "large" branches, whereas a smaller time step is used to step computations forward on "small" branches within the large time step.

An initial calibration of the model to 1975 data has been completed with encouraging results. Similar results from an application using 1976 data with no additional calibration of the model are also presented.

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PREFACE

The work described herein and the preparation of this report were conducted during the period March 1974 to July 1976 for the U. S. Army Engineer Division, Ohio River (ORD), by the U. S. Army Engineer Waterways Experiment Station (WES) under the general supervision of Messrs. H. B. Simmons, Chief of the Hydraulics Laboratory, and M. B. Boyd, Chief of the Mathematical Hydraulics Division (MHD).

Dr. B. H. Johnson, MHD, conducted the study and prepared this report. Mr. Ron Yates of ORD aided in the data collection.

Directors of WES during the conduct of this study and the preparation and publication of this report were COL G. H. Hilt, CE, and COL John L. Cannon, CE. Technical Director was Mr. F. R. Brown.

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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
feet	0.3048	metres
miles (U. S. statute)	1.609344	kilometres
cubic feet per second	0.02831685	cubic metres per second

A MATHEMATICAL MODEL FOR UNSTEADY-FLOW COMPUTATIONS THROUGH
THE COMPLETE SPECTRUM OF FLOWS ON THE LOWER OHIO RIVER

PART I: INTRODUCTION

Purpose and Scope

1. The U. S. Army Engineer Division, Ohio River (ORD) is responsible for operation of the navigation dams on the Ohio River to maintain a navigable channel during low-flow periods. In addition, during periods of flooding on the lower Ohio and lower Mississippi Rivers, ORD directs the operation of Barkley and Kentucky Reservoirs on the Cumberland and Tennessee Rivers, respectively. The primary objectives of flood control regulation by these reservoirs are to:

- a. Safeguard the Mississippi River levee system.
- b. Reduce the frequency of use of the Birds Point-New Madrid floodway.
- c. Reduce the frequency and magnitude of flooding of lands along the lower Ohio and Mississippi Rivers that are unprotected by levees.

It is obvious that a mathematical model capable of accurately predicting Ohio River stages during periods of flooding would be extremely useful in determining the most efficient reservoir operation plan at Barkley and Kentucky Reservoirs for a given flow on the Ohio and Mississippi Rivers. In addition, if such a model included the effect of the navigation locks and dams it would be of great use in establishing more efficient and systematic gate manipulations along the river during low-flow periods. Modification and subsequent calibration of an existing mathematical model, SOCHMJ (Simulation of Open Channel Hydraulics in Multi-Junction Systems), to provide ORD with such a capability were the objectives of the project described herein.

2. A mathematical model capable of providing ORD with the capability discussed above along the Ohio River from McAlpine Locks and Dam near Louisville, Ky., to the Ohio's junction with the Mississippi River at Cairo, Ill., involves the calculation of unsteady flows in a system

composed of portions of the Ohio, Green, Wabash, Cumberland, Tennessee, and Mississippi Rivers. The equations which govern such flows are statements of the conservation of mass and momentum of the flow field and may be written as:

$$\text{Continuity: } \frac{\partial h}{\partial t} + \frac{1}{B} \frac{\partial (AV)}{\partial x} - \frac{q}{B} = 0 \quad (1)$$

$$\text{Momentum: } \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial h}{\partial x} + \frac{qV}{A} + \frac{gn^2 V |V|}{2.21R^{4/3}} = 0 \quad (2)$$

where

h = water-surface elevation above mean sea level*

$\partial/\partial t$ = rate of change with respect to time

B = width of water surface

A = cross-sectional flow area

V = mean flow velocity

$\partial/\partial x$ = rate of change with respect to distance

q = lateral inflow per unit distance along the channel per unit time

g = acceleration due to gravity

n = Manning's resistance coefficient

R = hydraulic radius

These equations are often referred to as the equations of St. Venant. A brief discussion of their properties and the assumptions underlying their derivation are presented in any open channel hydraulics text and are summarized in Reference 1.

3. The attenuation or amplification of unsteady flows such as flood waves is controlled by inertial, friction, and pressure terms in the momentum equation, and by the mechanism of storage in the channel and its overbanks. In the past, the momentum equation has been simplified by omitting some of the terms so that solutions could be more easily obtained. Early models completely neglected the dynamic or momentum equation to yield what are commonly referred to as storage

* For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix B).

routing models. An approximation of the complete dynamic equation in which the pressure and inertial terms are neglected results in the "kinematic wave" model. Similarly, if the inertial terms are neglected but the pressure term is retained the "diffusion wave" model results. Incorporating the full momentum equation yields the complete solution, which is often labeled the "dynamic wave" model. These different mathematical models and their solutions are discussed in Reference 2. A summary of the various terms retained in these models is presented below:

Continuity Equation

$$\frac{\partial h'}{\partial t} + v \frac{\partial h'}{\partial x} + h' \frac{\partial v}{\partial x} = 0 \quad (3)$$

Storage Routing

Momentum Equation

$$\left| \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial h'}{\partial x} \right| S_o + \frac{gn^2 V |V|}{2.21 R^{4/3}} = 0 \quad (4)$$

Kinematic Wave
 Diffusion Wave
 Dynamic Wave

where S_o is the bottom slope of channel. Fread³ has determined the significance of the various terms in the dynamic equation for both a gradually and rapidly varying transient on the lower Mississippi River (Table 1). For the case of the slowly varying transient (rise of 1.5 ft/day* at RM 138.7), the local acceleration term is negligible compared with the friction slope term. However, the convective acceleration term is about 22 percent of the friction slope while the pressure or water slope term has a magnitude of about 80 percent of the friction slope. From these results, it is obvious that at least the diffusion wave model approximation and preferably the complete dynamic wave model

* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 3.

should be employed for flood computations on the lower Mississippi River. Based upon other results from Fread,⁴ it can be determined that the dynamic looping effect in stage-discharge plots on a river such as the Ohio with a slope of 0.00005 ft/ft and a rate of rise of the water surface on the order of 4.0 ft/day is about +2.0 ft. Also, when calculating unsteady flows which are significantly influenced by upstream as well as downstream conditions, such as a lock and dam or a junction of a major tributary with the main river, the most accurate modeling approach is to utilize the complete continuity and momentum equations.

Background

4. Since 1971, the U. S. Army Engineer Waterways Experiment Station (WES) has been involved in the computation of unsteady flows on the Ohio River. At its twenty-seventh meeting on 19 May 1970, the Mississippi Basin Model Board (MBMB) approved a study to develop computer programs for unsteady-flow computations along reaches of the Mississippi River and its larger tributaries. At the thirty-second meeting on 7 January 1971, in a joint effort with ORD, the area of responsibility of WES was determined to be the lower Ohio River from Louisville, Ky., to its junction with the Mississippi River. This initial study⁵ involved the application of two mathematical models developed by the Tennessee Valley Authority (TVA).⁶ Although the accuracy of the results from this study left much to be desired, the study did demonstrate the feasibility of solving numerically the complete St. Venant equations applied to a large and complicated system such as the Ohio River and its major tributaries.

5. At the conclusion of the MBMB study, it was decided that a need existed for a model capable of predicting stages accurately at Cairo for given release schedules at Barkley and Kentucky Reservoirs. To provide ORD with such a model, the previously applied TVA model, SOCHJ, was reprogrammed and modified to handle a system composed of an unlimited number of rivers.¹ This was required due to the necessity of treating the Cumberland and Tennessee Rivers as dynamic branches of the system. In addition, more accurate geometric data were collected from

the Mississippi Basin Model. The physical limits of the modeling effort to provide ORD with a model to aid in planning releases at Barkley and Kentucky Reservoirs are shown in Figure 1. As illustrated in Figure 2,

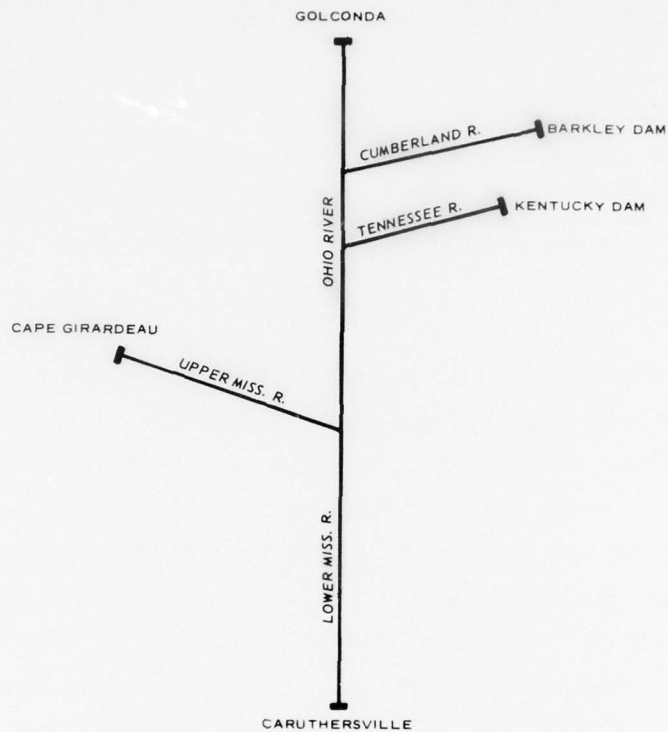


Figure 1. Model limits of previous application of SOCHMJ

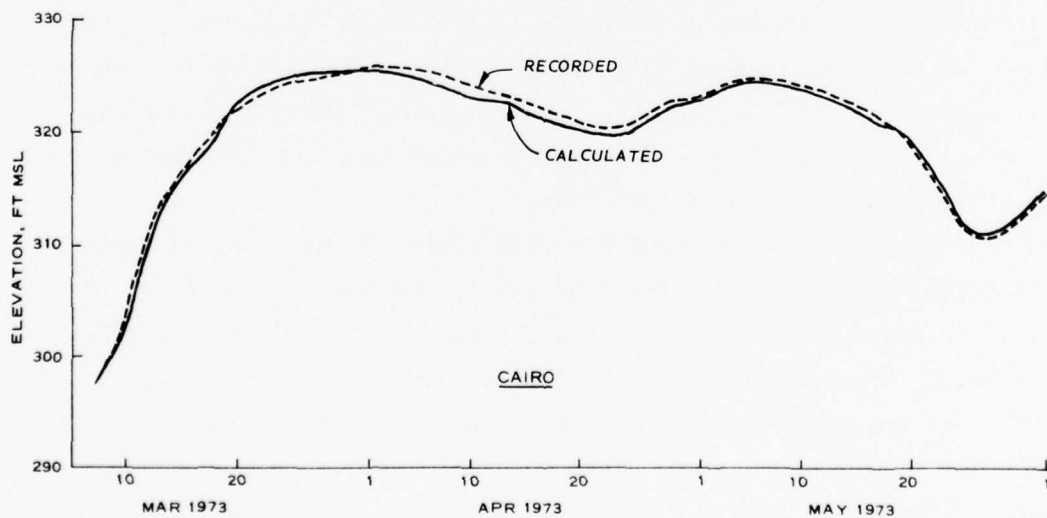


Figure 2. 1973 elevation hydrographs at Cairo, Ill.

results from this study were extremely encouraging. Since the latter part of 1974, ORD has used the model SOCHMJ to make daily forecasts at Cairo as well as at other points along the Ohio and Mississippi Rivers.

6. After applying SOCHMJ with great success, ORD then expressed an interest in extending the model to Louisville, Ky., with the requirement that the effect of the high-lift locks and dams on the Ohio River be included in the solution. In addition, the Green and Wabash Rivers were to be treated as dynamic branches of the system, along with the Cumberland and Tennessee Rivers. Figure 3 illustrates the limits of the physical system to be modeled and the locations of the locks and dams on the Ohio which are to be included.

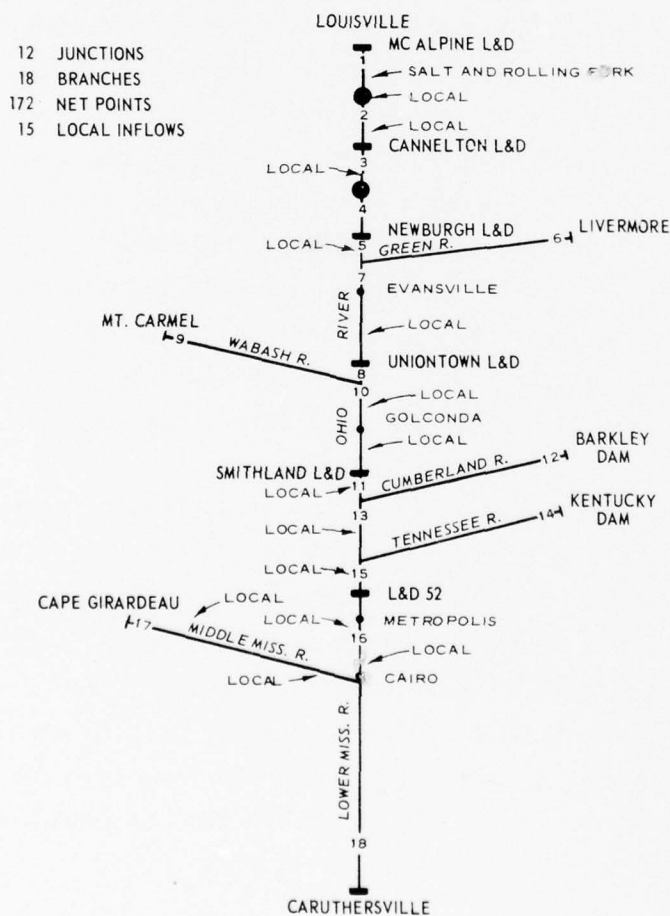


Figure 3. Physical limits of current application of SOCHMJ

PART II: NUMERICAL MODEL DEVELOPMENT

Discussion of Basic Model - SOCHMJ

7. As previously noted, the TVA unsteady-flow model has been modified and recoded, and the resulting model is called SOCHMJ. The modified model can be applied to a system composed of an unlimited number of junctions and branches, including a system containing no tributaries, i.e., a single river. Also, the model can be applied to a system such as encountered in the flow around an island. In addition to the FORTRAN coding and applicability to multi-junction systems, one other major difference exists between the original TVA model and SOCHMJ. In SOCHMJ, Manning's n is allowed to vary with elevation as well as with distance along the channel, whereas the original TVA model allows only for variation with distance along the channel.

8. Although, as indicated above, the coding has been altered to provide for a more understandable computer program, the same centered explicit finite difference scheme developed by Stoker⁷ under an ORD contract and employed in the TVA model has been utilized in SOCHMJ for the numerical solution of the St. Venant equations. With such a computation scheme, the solution on a particular time line can be directly determined at each net point. In other words, the solution marches forward in time from one time line to the next.

9. As discussed above, SOCHMJ can be applied to a system composed of many rivers, with each river labeled as a branch of the system. In the discretization of the physical system, a restriction on the number of Δx 's per branch exists. A minimum of four Δx 's must be contained within each branch with the additional restriction that if more than four exist, the total must be an even number. On each branch, all Δx 's have an equal length, although the length can vary from one branch to another. However, the time step Δt prescribed is common to all branches. Thus, since the stability criterion

$$\left(v + \sqrt{g \frac{A}{B}} \right) \Delta t / \Delta x \leq 1 - \frac{gn^2 |v| \Delta t}{2.21R^{4/3}} \quad (5)$$

must always be satisfied, it is obvious that the smallest Δx in the system determines the Δt to be used for computations on all branches. It is then easy to see that one small branch in a large system can result in a very inefficient operation of the model. As will be discussed later, it was necessary to relax this restriction, although it has not been completely removed, to make the application of SOCHMJ to the system illustrated in Figure 3 economically feasible. Application to the system shown is called the Ohio problem in the remaining discussion.

Modifications Required for the Ohio Problem

The physical system

10. As illustrated in Figure 3, a system composed of six rivers with a total of approximately 700 river miles is to be modeled. In addition, the effects of five locks and dams, namely Cannelton, Newburgh, Uniontown, Smithland, and Lock and Dam 52, are to be accounted for in the flow computations. In order to handle the internal flow control structures more efficiently and in a straightforward manner, each lock and dam is considered to be a junction consisting of two branches. In addition, since the model does not allow for the specification of boundary conditions at both ends of a branch, it was necessary to create artificial junctions between Louisville and Cannelton Locks and Dam and between Cannelton Locks and Dam and Newburgh Locks and Dam. Figure 3 shows that the system is then composed of a total of 12 junctions and 18 branches. The discretization of the system is presented in Table 2.

Lock and dam computation procedure

11. During low-flow periods, the various navigation dams on the Ohio River are operated to maintain a 9-ft navigation channel. This is accomplished by manipulating the gates to maintain a particular elevation upstream of the dam, i.e., the flow is controlled. On the rising side of a flood wave, the gate operator maintains the desired elevation

by raising more gates. When all the gates are finally removed, the flow field becomes that of essentially a free or uncontrolled river. As the peak passes and the recession side of the wave is encountered, more and more gates are lowered in order to maintain the desired navigation depth and the river is once more in a controlled state.

12. As discussed in Reference 1, the governing differential equations constitute a hyperbolic system. Thus, as long as the flow is subcritical, either the water-surface elevation or the discharge or perhaps a rating curve at a downstream boundary must be prescribed at an open boundary. The mathematical model SOCHMJ operates such that when one dependent variable is prescribed, the other is calculated at the boundary. Therefore, the lock and dam problem is handled in the following manner. The downstream boundary condition of the branch immediately upstream of a lock and dam is prescribed to be the elevation the operator attempts to maintain, so long as the flow is being controlled. The discharge corresponding to the fixed elevation is then computed and used as the upstream boundary condition of the branch immediately downstream of the lock and dam. The downstream elevation corresponding to the prescribed discharge is then computed and compared with the prescribed upstream elevation. If the downstream elevation is less than the upstream elevation, the flow is controlled to some degree; however, when the two elevations become equal, all gates are out of the water and the river is in an uncontrolled state. At this point, the lock and dam is assumed to be a junction. As with the junction of two rivers, computations are then made to ensure that the elevations of the branches are equal at the junction. Details of the computation cycle are presented later.

Small branch computation procedure

13. As noted in previous discussion, a restriction imposed by SOCHMJ is that each branch must be subdivided into at least four Δx 's. As indicated in Table 2, this forces the Ohio problem to contain a Δx of 0.60 mile on branch 8 between Uniontown Locks and Dam and the junction of the Ohio and Wabash Rivers. With a spatial step of 0.60 mile, the stability criterion (given in Equation 5) forces the Δt for the complete system to be on the order of 60 sec. However, on a majority of

the branches the discretization is such that a Δt on the order of 300 sec can easily be specified with stable computations maintained. Since the restriction of four Δx 's per branch could not be removed due to the computation procedure at the junctions, another solution to the time-step problem had to be found.

14. The approach taken was the specification of two time steps, one to apply to small branches such as branches 5, 8, 11, and 15 and a larger Δt to be applied to the remaining 14 branches. As illustrated in Figure 4, the large Δt must be an odd multiple of the small time

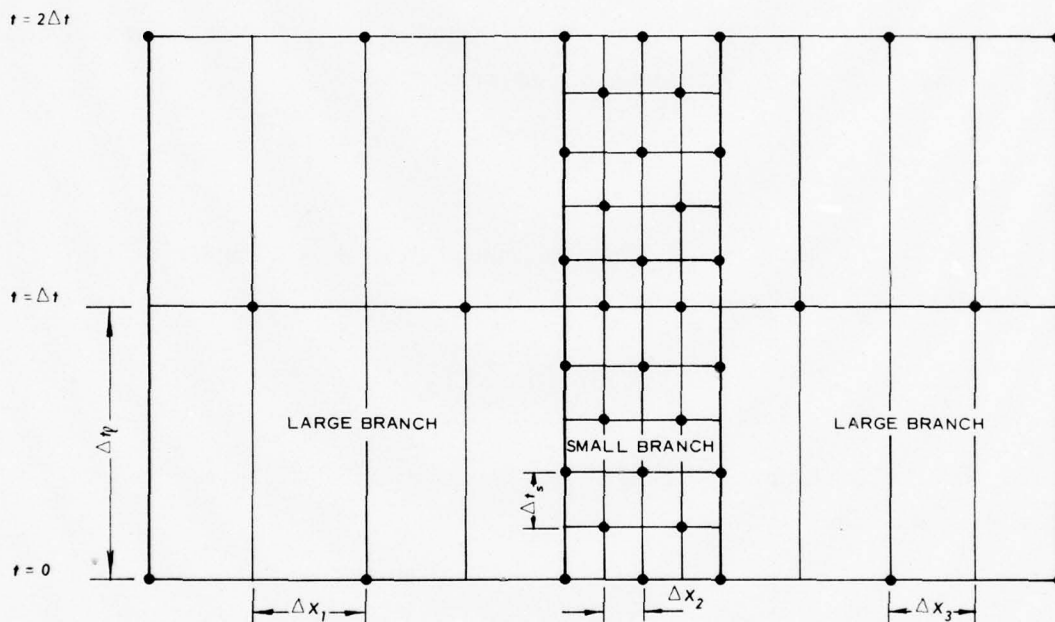


Figure 4. Two time-step option

step. Computations on small branches are made at small time-step intervals within each large time step. Such computations within a large time step are repeated until the elevations at the junctions of the large and small branches converge within a specified limit. Details of the computation cycle are presented in a later section.

15. As indicated above, iterations over the small branches within each large time step occur. Thus, it would appear that in some cases less computer time might be required if the small time step were used

for all branches rather than using the two time-step option. An attempt to provide guidance on the feasibility of the small branch option is presented below:

Let

- L = number of net points on large branches
- S = number of net points on small branches
- Δt_l = time step for large branches
- Δt_s = time step for small branches
- N = average number of iterations required for computations on the small branches within each large time step

Without the option of specifying small branches, the number of net point computations per large time step is approximately:

$$(L + S) \left(\frac{\Delta t_l}{\Delta t_s} \right)$$

whereas with the small branch option, the number of computations per large time step can be expressed

$$L + S \left(\frac{\Delta t_l}{\Delta t_s} \right) N$$

Thus, for the small branch option to be economically feasible the following must hold:

$$(L + S) \left(\frac{\Delta t_l}{\Delta t_s} \right) > L + SN \left(\frac{\Delta t_l}{\Delta t_s} \right) \quad (6)$$

or, after rearranging terms,

$$\frac{L}{S} > \frac{\left(\frac{\Delta t_l}{\Delta t_s} \right) (N - 1)}{\frac{\Delta t_l}{\Delta t_s} - 1} \quad (7)$$

Assuming $\Delta t_l / \Delta t_s = 5$ and based upon sample runs $N = 3$, the ratio of total net points on large branches to the number on small branches must be greater than 2-1/2 for the small branch option to be viable. Table 2

shows that this ratio is 8.2 for the Ohio problem and thus the use of two time steps should result in significant savings in computer time.

Input Data Required

16. The modifications of SOCHMJ required for application to the Ohio problem illustrated in Figure 3 have been made in such a manner that the model retains its general applicability to multi-junction systems, although the lock and dam procedure, as currently coded, can only be applied to the particular physical system shown in Figure 3. The option of classifying branches as either large or small branches and employing two time steps is available for all applications.

17. Data required for the operation of SOCHMJ are read from cards. The first data card contains basic information such as the total number of net points in the system, the total number of junctions and branches, the number of stations at which output is desired, and the large time step to be employed. The second group of data contains information about the stations at which output is desired. In addition to requesting output at particular stations, output can be requested at any of the net points of the system. The third group of data contains information about each branch such as the type of boundary condition prescribed, the size of the spatial step, and whether the branch is a large or small branch. The fourth data group contains information about the junctions, e.g., the numbers of the branches associated with each junction. The next major data group consists of the tables of geometric data. A table of top width, flow area, (hydraulic radius)^{2/3}, and Manning's n , all as functions of elevation, must be input at each net point of the system. A more detailed discussion of the geometric data for the Ohio problem is presented later. The next data group consists of the initial values of the elevation and discharge at all grid points on the first two time lines. The specification of initial conditions is flexible due to the characteristic of hyperbolic equations that the solution becomes independent of initial conditions after a sufficient length of time. The final major data group required by SOCHMJ consists of the time-dependent

boundary conditions which must be prescribed at each open boundary and any lateral inflows into the system. A detailed listing of the data required by SOCHMJ in its present (June 1977) form is presented in Appendix A.

Computation Cycle for Ohio Problem

18. As previously noted, SOCHMJ utilizes an explicit finite difference scheme for the numerical solution of the St. Venant equations. With such an explicit scheme, the solution on a particular time line can be directly determined at each net point, i.e., the solution marches forward in time from one time line to another. The computations performed from one time line to the next are referred to as the computation cycle and are outlined for the Ohio problem in the following steps:

<u>Steps</u>	<u>Action Taken</u>
1	Various constants and variables are initialized.
2	All input cards except the boundary conditions are read. The Ohio problem is set up as if the dams are junctions.
3	Before any computations are made, subroutine PRSET is called to set up the proper problem, i.e., which dams are controlling the flow. Dams that are controlling the flow have B.C.'s set immediately upstream.
4	Computations at interior points of first large branch are made.
5	If the large branch for which interior computations have just been made contains a boundary, boundary computations are made.
6	Steps 4 and 5 are repeated for all large branches. If the Cannelton Dam is controlling the flow, the calculated Q at the downstream end of branch 3 is set as the upstream B.C. of branch 4 before any computations are made for branch 4.
7	Subroutine SMALL is called.

(Continued)

<u>Steps</u>	<u>Action Taken</u>
8	If this is the first iteration, i.e., the first time SMALL has been called for this particular time line, values on the previous two time lines are saved as initial conditions to be used for each successive time SMALL is called during the current large time step.
9	If this is not the first time SMALL has been called during this time step, initial conditions are reestablished from the result of step 8.
10	Computations at the interior points of the first small branch are made.
11	If this small branch is downstream of a lock and dam controlling the flow and the time line is a boundary time line, upstream boundary computations are made with discharges prescribed from the branch upstream of the dam.
12	If this small branch is part of a normal junction at its upstream end, boundary computations are made by linearly approximating the elevation at the junction at the end of the current large time step if this is the first time SMALL has been called during this large time step. If this is not the first time SMALL has been called, the calculated junction elevation is used as the upstream B.C. for this small branch.
13	Step 12 is repeated for the downstream end of this small branch.
14	Steps 10-13 are repeated for each of the small branches.
15	Step 10 begins again for the next small time step. This looping action continues until the end of the large time step is reached. There must be an odd number of small time steps within the large one.
16	The small branch values are reindexed so that the time indices will match those of the large branches.
17	Control is now transferred back to the main program and computations at all normal junctions are made.

<u>Steps</u>	<u>Action Taken</u>
18	If ITER = 1 , i.e., SMALL has only been called once, ITER is incremented by 1 and SMALL is called again. Steps 7-17 are repeated. If ITER > 1 , a check is made at each junction. If the difference between the elevation now and after the previous value of ITER is greater than DSI, SMALL is called again. Steps 7-18 are repeated. If the difference at each junction is less than DSI, the time is updated and a check is made for printing of output.
19	Steps 3-18 are repeated.
20	When new boundary conditions cease to be read in, the program terminates.

Output Provided

19. Output can be obtained from SOCHMJ at specified stations or river miles as well as at all or specific net points. At such net points, output in the form of elevations, velocities, and discharges plus geometric data if desired, is provided. At particular river mile locations where output is requested, similar information except for geometric data is printed. Output is provided after a certain interval of time steps, the value of which is variable and is specified in the input data.

PART III: MODEL CALIBRATION AND PRESENTATION OF RESULTS

20. Field data recorded during 1975 were used in the initial calibration of the model described herein. A more accurate calibration through a wider spectrum of flow could not be accomplished since Uniontown and Smithland Locks and Dams were still under construction. The time-dependent conditions specified at the external model boundaries were as follows:

- a. Discharges were specified at Louisville on the Ohio.
- b. Discharges were specified at Livermore on the Green.
- c. Discharges were specified at Mt. Carmel on the Wabash.
- d. Discharges were specified at Barkley on the Cumberland.
- e. Discharges were specified at Kentucky on the Tennessee.
- f. Discharges were specified at Cape Girardeau on the upper Mississippi.
- g. A rating curve in the form of a table of elevations versus discharges was specified at Caruthersville on the lower Mississippi.

As previously discussed, during periods when a particular lock and dam controls the flow, a constant elevation is set as an internal boundary condition upstream of the lock and dam.

21. The boundary hydrographs are presented in Plate 1. Table 3, which was obtained from data provided by the U. S. Army Engineer District, Memphis, presents elevations and corresponding discharges used as the boundary condition at Caruthersville. The looped nature commonly observed in actual rating curves is not allowed in SOCHMJ.

Initial Conditions

22. As noted previously, initial values of elevation and discharge must be specified at each net point on the first two time lines. During the calibration phase of a mathematical modeling study, values are known initially at various points in the system from historical data. Values at intermediate net points are then specified either by a "best guess" or perhaps interpolation. Runs to assess how rapidly

errors in initial conditions dampen revealed that even though the initial state of the system was changed significantly, essentially identical results were obtained after three or four days of computations using a large time step of five minutes. As previously discussed, the St. Venant equations constitute a hyperbolic system and as such possess the characteristic that the effect of initial conditions diminishes as time progresses.

Geometric Data

23. SOCHMJ requires as input at each net point a table of geometric data consisting of flow area, top width, (hydraulic radius)^{2/3}, and Manning's n versus water-surface elevation. In earlier studies,^{1,5} the system illustrated in Figure 3 was divided into small reaches on the Mississippi Basin Model and storage volume data were collected for each reach. As discussed in Reference 5, these data were then converted to the tables described above. In addition, channel and overbank cross sections were constructed at many points in the system from hydrographic and topographic maps; tables were then constructed from these for use in areas where uncertainty of the accuracy of the storage volume data existed. In this and earlier studies involving SOCHMJ, overbank areas are considered as only storage areas and thus the flow area in the geometric tables does not include cross-sectional area on the overbanks.

Calibration Procedure

24. With the spatial steps shown in Table 2 employed for the 18 branches of the system, a large time step of 300 sec and a small time step of 60 sec were found to yield stable computations. Calibration of the model was accomplished by comparing calculated and recorded values of elevation and discharge at several points in the system. The calibration or "matching" of elevations was accomplished by varying the values for Manning's n at net points in the neighborhood of the check-point. Increasing n at upstream stations decreases the elevation,

whereas increases at downstream stations increase the elevations. Discharges were brought into agreement by changing the n values at all net points on a particular branch by the same amount. Lateral inflow data were furnished by ORD as the total estimated inflow within large reaches of the Ohio River, e.g. Louisville to Evansville. These were subdivided within each large reach and input in equal amounts at the various local inflow points illustrated in Figure 3. The lateral inflow hydrographs are presented in Plate 2. Results from initial model runs indicated that the estimated lateral inflow from Louisville to Evansville was probably too high. Therefore, the Louisville-Evansville inflow hydrograph shown in Plate 2 was reduced by 50 percent, a value that is hydrologically acceptable, from 30 March through 4 April in the calibration results to be presented in the following discussion.

Calibration Results

25. Plates 3 and 4 illustrate the comparison of calculated and field values of elevations at McAlpine (Louisville, Ky.), Cannelton, Newburgh, Uniontown, and Smithland Locks and Dams, and Cairo on the Ohio River. From an inspection of these plots, it is observed that in most cases agreement is better on the rising side and at the peak of the hydrograph than on the recession side. This problem could perhaps be alleviated by allowing Manning's n to vary, depending upon whether the water-surface elevation is rising or falling. However, no justification for such a variation is known; therefore SOCHMJ was not modified to allow for this type of variation in Manning's n . Two possible explanations for the above inconsistency in hydrograph reproduction are as follows. First, it seems reasonable to believe that perhaps as the river falls, the flow of the water from the overbank areas into the river channel adds significant momentum to the channel flow which is not accounted for in SOCHMJ. This would then result in a higher wave velocity on the recession side than that calculated by the mathematical model. In addition, many substantial depressions exist in the floodplains that retain water as the overbank areas drain into the river channels, whereas the

mathematical model assumes that all of the water on the overbank moves back into the channel. Plate 5 illustrates the larger calculated discharge on the recession side at Evansville, Ind.

26. Again, it should be noted that the calibration results presented herein were obtained using flows such that the partially completed Uniontown and Smithland Locks and Dams were never called upon to control the flow. After these structures are completed, additional calibrations should be undertaken.

Results from Prescribing Elevations as the Boundary Condition at McAlpine Locks and Dam

27. Plate 3 reveals a rather substantial difference in computed and recorded elevations at McAlpine Locks and Dam from 6 April through 13 April. Therefore, it was decided to apply the calibrated model to the 1975 flood using recorded elevations rather than discharges as the upstream Ohio boundary condition at McAlpine Locks and Dam. Results from this run are included in the calibration results presented in Plates 3 and 4. Use of the more accurate recorded elevations as the boundary condition results in substantially better matching at the lower end of the hydrographs at Cannelton and Newburgh Locks and Dams, with little difference in the computations observed at downstream points. However, since in an operational mode the model will normally be run with forecast discharges, the initial calibration previously discussed was conducted with discharges as the upstream boundary condition.

Results from Application to 1976 Data

28. An application of the calibrated model has been made using 1976 data furnished by ORD. In this application, the Uniontown Dam as well as the Cannelton and Newburgh Dams controlled the flow for a substantial portion of the period during which unsteady-flow computations were made. Elevations were prescribed as the boundary condition at McAlpine Locks and Dam with discharges prescribed at the open boundaries of the Green, Wabash, Cumberland, Tennessee, and upper Mississippi Rivers

(Plate 6). Lateral inflows were prescribed at the same locations as noted in Figure 3. A comparison of computed and recorded elevations downstream of the Cannelton, Newburgh, and Uniontown Locks and Dams and at Cairo, Ill., are presented in Plate 7. Except for the very low flow portions of the Cannelton and Cairo hydrographs, the results are satisfactory considering the limited calibration experienced by the model. Actually, it should be pointed out that essentially no calibration has been attempted at these low flows since the calibration with the 1975 data did not cover such flows. It might also be noted that the effect of inaccurate initial conditions is probably reflected in the computations for longer periods of time when low flows are experienced immediately after initiation of the computations. Also, with a rate of change of elevation of 10.0 ft/day such as occurs between 18 and 19 February at McAlpine Locks and Dam, daily input of boundary conditions with a linear interpolation to provide values at the end of each time step is probably insufficient for the accuracy desired. Plate 8 presents a comparison of computed and recorded 1976 discharges at Evansville, Ind.

PART IV: CONCLUSIONS AND RECOMMENDATIONS

29. In this study, numerical solutions have been obtained of the open channel unsteady-flow equations applied to a system composed of portions of six rivers, namely the Ohio, Green, Wabash, Cumberland, Tennessee, and Mississippi Rivers, which take into account the effect of five locks and dams on the Ohio River. The modeling limits were McAlpine Locks and Dam on the Ohio, Livermore, Ky., on the Green, Mt. Carmel, Ind., on the Wabash, Barkley Dam on the Cumberland, Kentucky Dam on the Tennessee, Cape Girardeau on the upper Mississippi, and Caruthersville on the lower Mississippi. Due to economic considerations, the mathematical model utilized was modified to allow for the specification of two time steps, one for branches with spatial steps about five miles in length and the other for smaller branches with spatial steps about a mile or less in length.

30. The mathematical model was calibrated by application to 1975 field data provided by ORD. The calibration was accomplished by varying Manning's n with distance along the rivers, as well as with elevation at some points, until recorded and computed values of elevations and discharges compared favorably. A comparison of elevation plots shows that in general good agreement is realized, especially on the rising side and the peak of the various hydrographs. The one discharge plot presented also demonstrates this behavior. Similar results were obtained from an application of the calibrated model using 1976 data.

31. Based upon the results of this study, it is concluded that the mathematical model provides ORD with a potentially useful operational tool. It should be remembered, however, that additional calibration and verification of the model are needed. Particular attention should be directed toward comparing calculated and recorded discharges through the various navigation dams. Due to the lack of data, only elevations were compared in the current study.

32. An additional effort has been funded by ORD to mathematically model the complete Ohio River plus its major tributaries from Pittsburgh, Pa., to its mouth at Cairo, Ill. Therefore, to the approximately

700 miles of river modeled in this study, an additional 800-900 miles of river will be added. Also, an additional 14 locks and dams will be added to the system. As shown in Figure 5, an additional 29 junctions and 35 branches will be added to the current system being modeled. The number of additional Δx 's will probably be in the neighborhood of 200 as compared with the 148 in the current application. For a run on the GE-635 computer located at WES, SOCHMJ applied to the system described in the present study requires approximately 2-1/2 hours

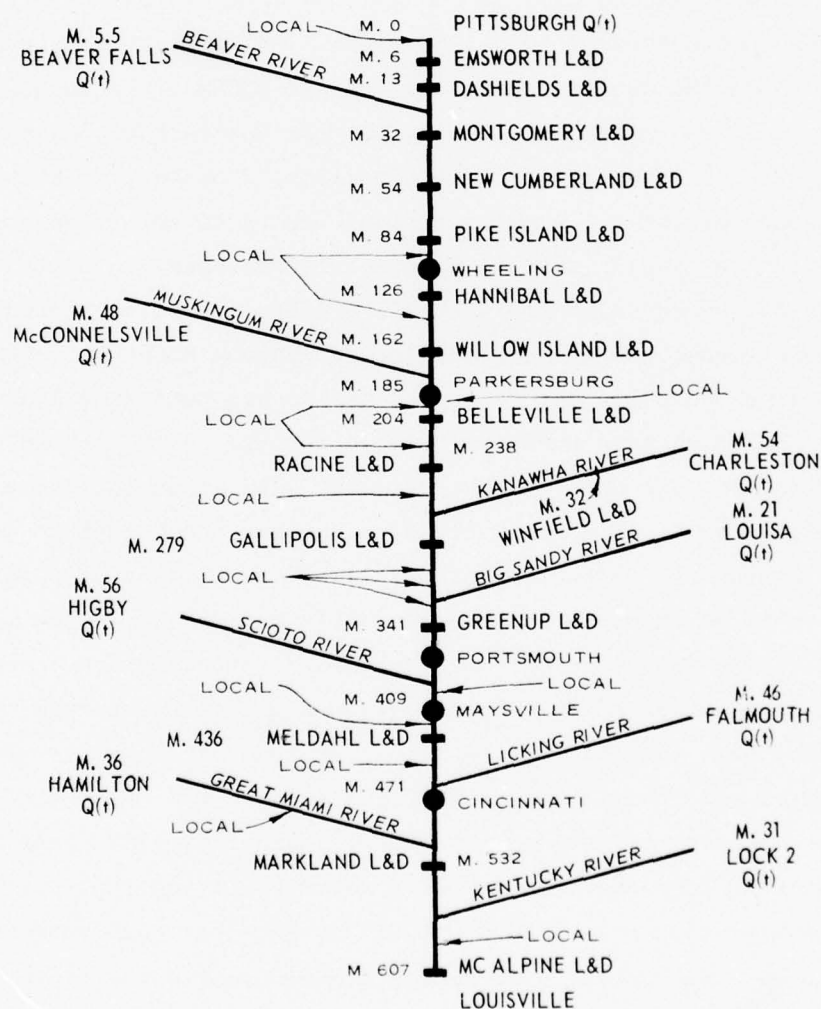


Figure 5. Proposed extension of model limits

of computer time for 30 days of flow computations. Assuming three iterations are required on each small branch, one finds the computer time per reach modeled per day of computations, λ , is approximately 1.56 sec/reach/day. Therefore, for the complete Ohio River system the computer time required on the GE-635 to simulate one day of flow would require at least 12 minutes. Actually, the time would be substantially greater since iterations at the additional junctions have been neglected in arriving at the above estimate. Obviously, the current application of SOCHMJ has pushed the model to its economic limit as an operational tool.

33. Both explicit, such as SOCHMJ, and implicit finite difference models of the unsteady-flow equations exist. In explicit solution procedures, such as the Stoker scheme utilized in SOCHMJ, the solution is obtained at each net point by progressing from upstream to downstream and the process is repeated for each time line. However, to achieve stable computations, i.e., small numerical errors do not increase in magnitude with succeeding computations, it is necessary to restrict the time-step size. When solutions are obtained by an implicit formulation, the difference equations are written for all points along the channel and solved simultaneously before proceeding to the next time line. Implicit computational procedures are stable for all Δt 's provided proper finite differences are used.⁸ However, even with an unconditionally stable implicit model it is easy to visualize how the operation of the locks and dams on the Ohio River will still force a restriction on the time step. The maximum time step permissible will be that time interval at which it is desired to check to determine if the dams are controlling the flow. It seems reasonable to expect such a check to be made at least twice a day. Therefore, the time-step restriction will probably be $\Delta t \leq 12$ hours. As previously noted, the large time step used in the current modeling effort is 5 minutes. Thus, it would appear that even though the locks and dams impose a restriction, an implicit model might use a time step more than 100 times larger than that currently being employed. Therefore, even though a more complicated computation procedure, an implicit formulation should seriously be considered in the modeling of the complete Ohio River and its major tributaries.

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Table 1
Comparison of Terms in Equation of Motion (After Fread³)

Location	Transient Flow Description	S_f ft/ft	S_o ft/ft	S_f/S_o %	$(1/g)(\partial v/\partial t)$ ft/ft	$(1/gS_f)(\partial v/\partial t)$ %	$(1/2g)(\partial v^2/\partial x)$ ft/ft	$(1/2gS_f)(\partial v^2/\partial x)$ %	$\partial h/\partial x$ ft/ft	$(\partial h/\partial x)(S_f)$ %
Reserve HM 138.7	1963 Flood Rising limb	0.0000130	0.0000014	930	0.0000003	2.0	-0.00000027	22	-0.0000106	82
Reserve FM 138.7	1963 Flood Peak	0.0000280	0.0000014	2000	-0.00000000	0.0	-0.00000063	22	-0.0000217	78
Carrollton FM 102.8	1969 Surge Rising limb	-0.00000028	0.0000014	200	-0.0000123	440.0	0.00000003	11	0.0000148	528
Carrollton FM 102.8	1969 Surge Peak	-0.00000032	0.0000014	228	-0.00000018	56.0	0.00000004	12	0.0000046	144

Table 2
Discretization of the Physical System

<u>Branch</u>	<u>Location</u>	<u>Δx, miles</u>	<u>No. of Δx's</u>
1	McAlpine Locks and Dam, RM* 320.9	4.675	12
2	RM 320.9, Cannelton Locks and Dam	5.017	12
3	Cannelton Locks and Dam, RM 233.0	4.617	6
4	RM 233.0, Newburgh Locks and Dam	4.617	6
5**	Newburgh Locks and Dam, Green River	2.025	4
6	Green River	5.933	12
7	Green River, Uniontown Locks and Dam	5.150	12
8**	Uniontown Locks and Dam, Wabash River	0.600	4
9	Wabash River	13.920	6
10	Wabash River, Smithland Locks and Dam	5.007	14
11**	Smithland Locks and Dam, Cumberland River	1.005	4
12	Cumberland River	4.597	6
13	Cumberland River, Tennessee River	3.320	4
14	Tennessee River	3.593	6
15**	Tennessee River, Lock and Dam 52	0.750	4
16	Lock and Dam 52, Mississippi River	5.325	8
17	Upper Mississippi River	5.237	10
18	Lower Mississippi River	4.992	24

* River miles above junction of Ohio and Mississippi Rivers.
 ** Small branches.

Table 3
Rating Curve Employed at Caruthersville*

<u>Elevation</u> <u>ft msl</u>	<u>Discharge</u> <u>cfs</u>
255.0	200,000
257.0	320,000
259.0	440,000
261.00	575,000
263.0	700,000
265.0	820,000
267.0	940,000
269.0	1,050,000
271.0	1,170,000
273.0	1,290,000
275.0	1,410,000
277.0	1,550,000
279.0	1,710,000
281.0	1,920,000
283.0	2,240,000
285.0	2,900,000

* Obtained from data provided by the U. S.
Army Engineer District, Memphis.

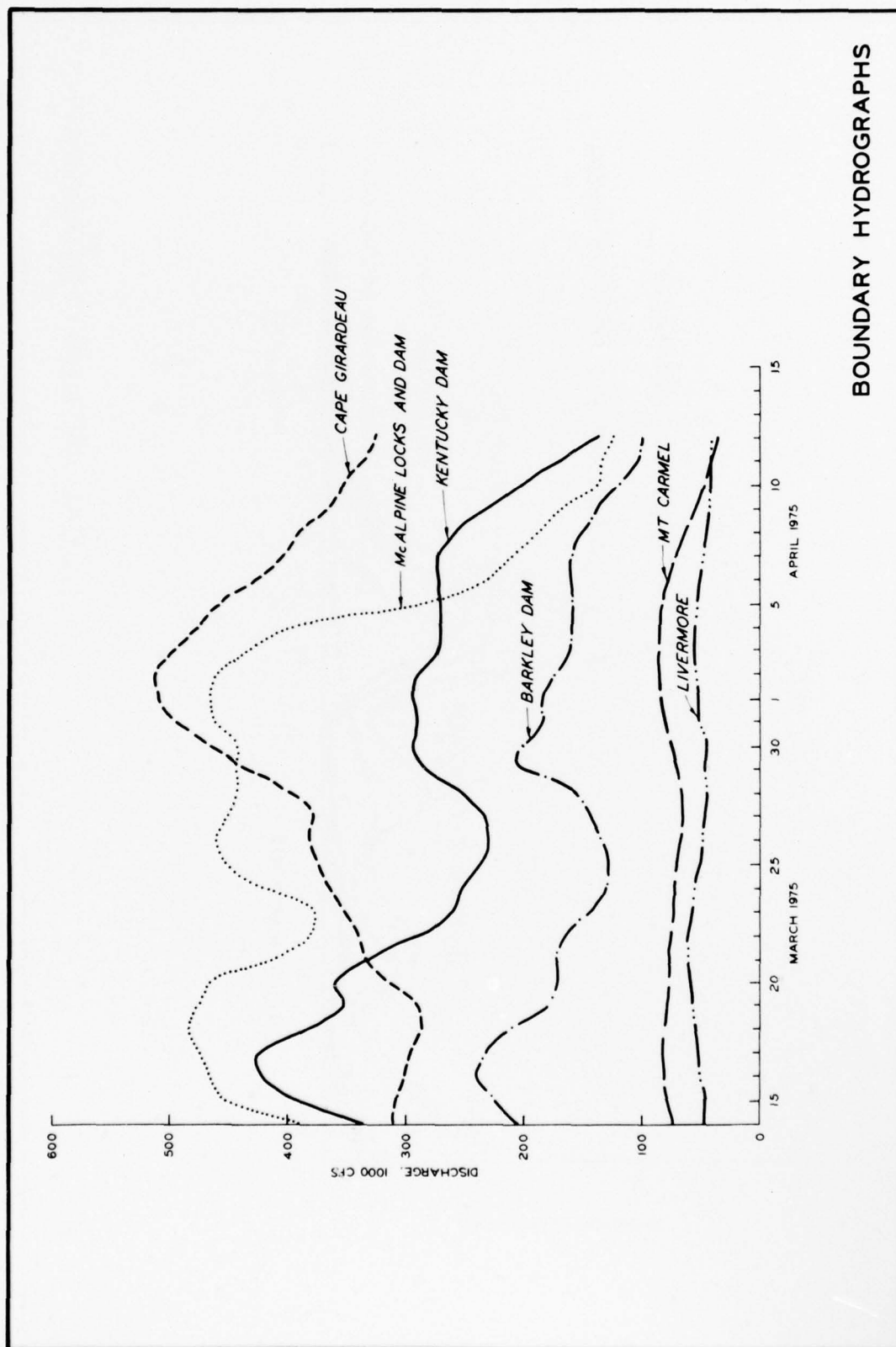


PLATE 1

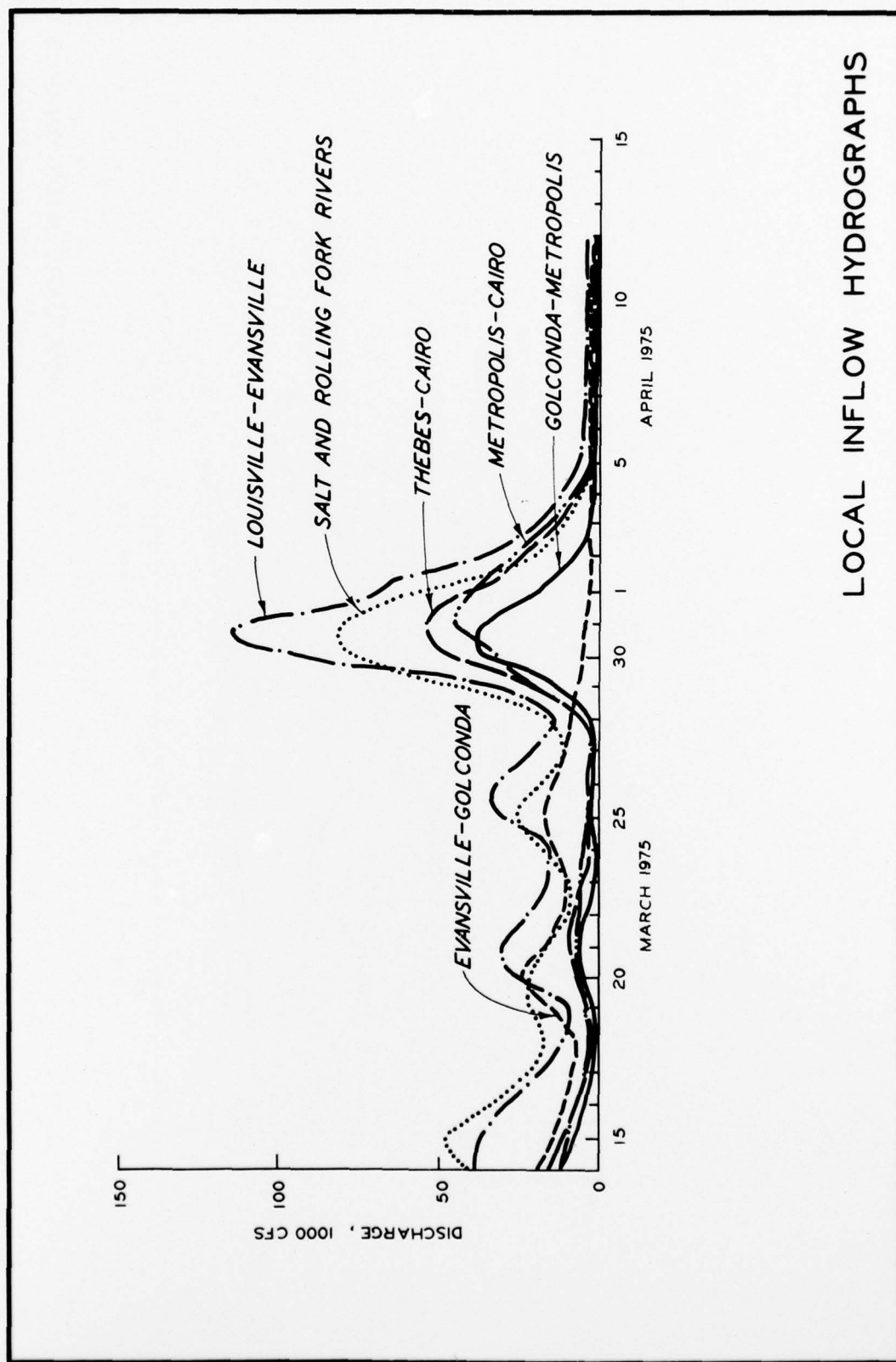
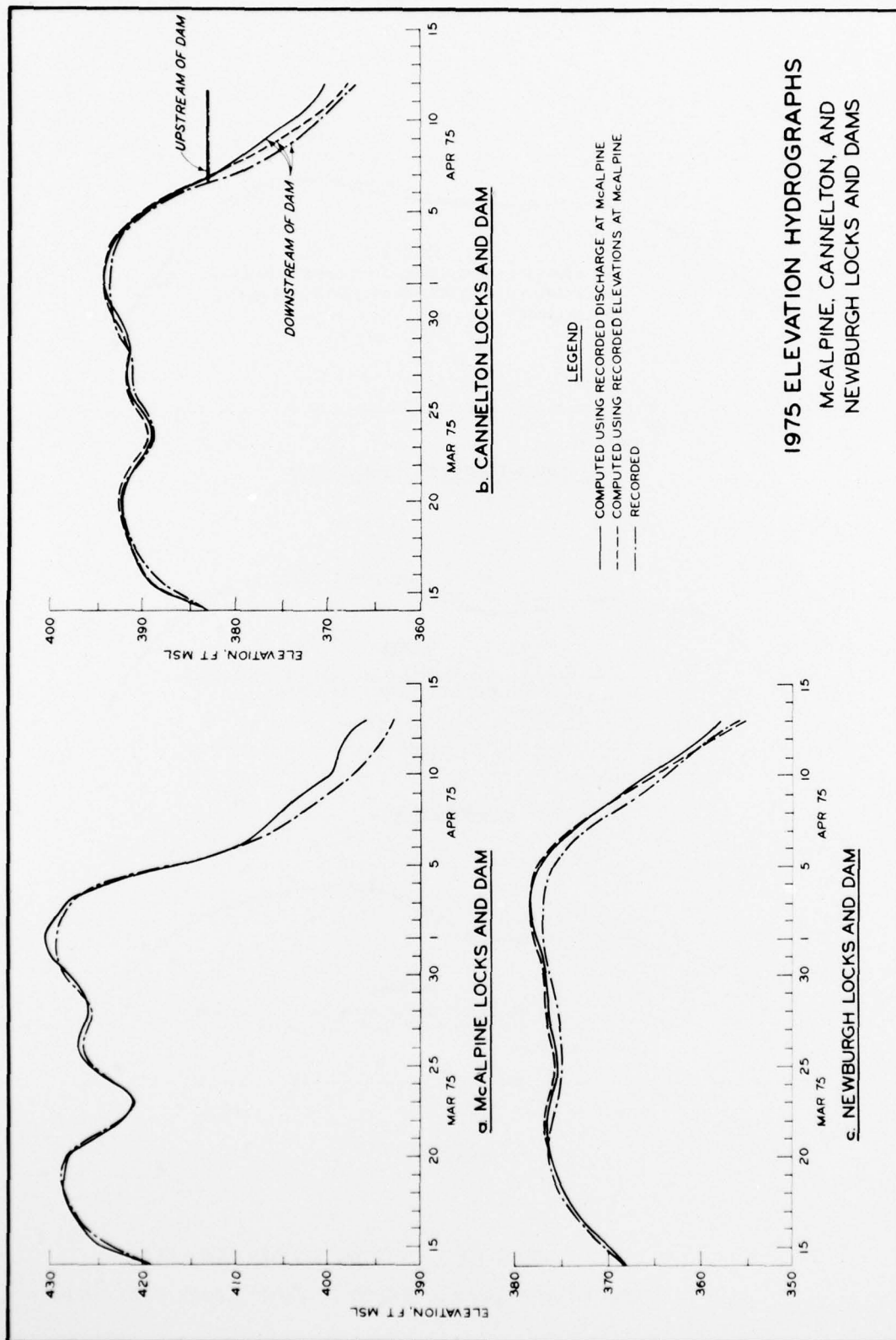
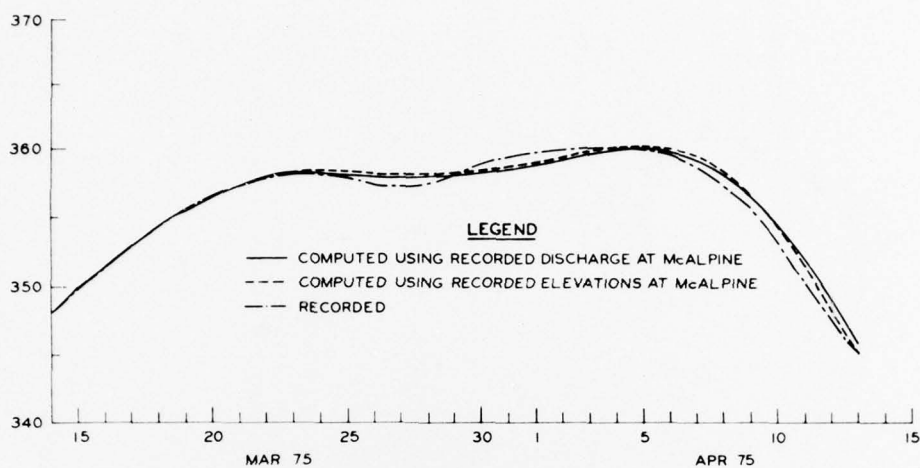
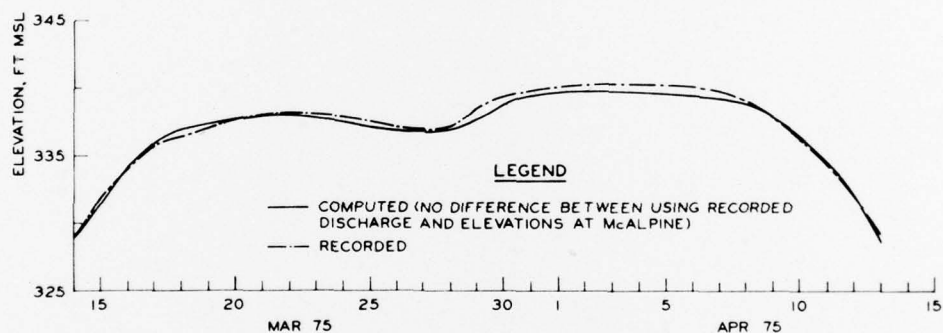


PLATE 2

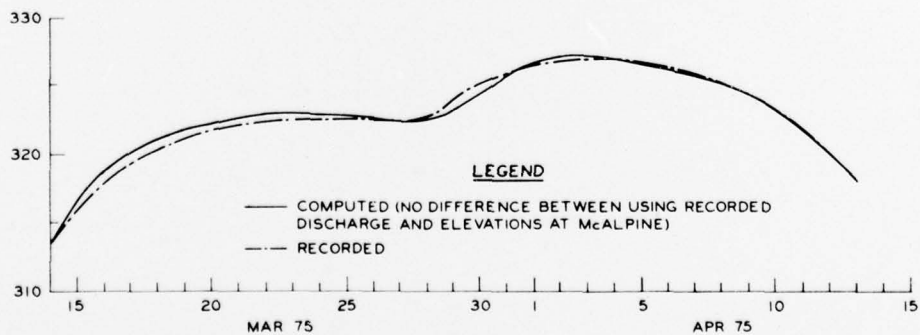




a. UNIONTOWN LOCKS AND DAM

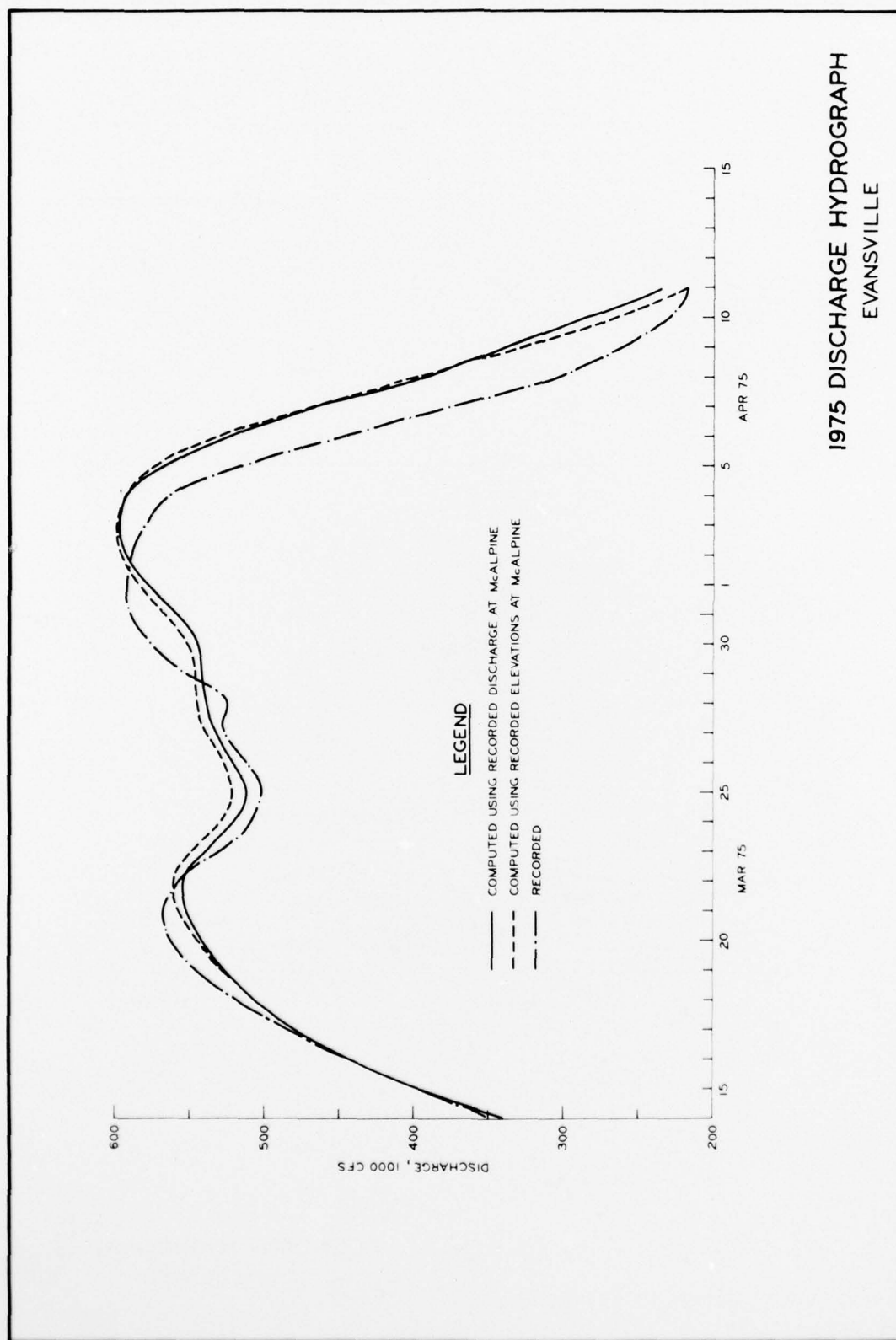


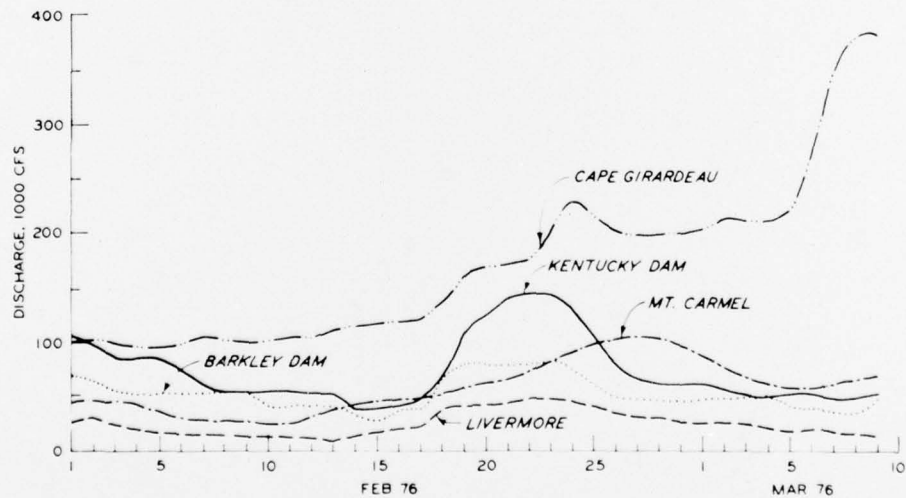
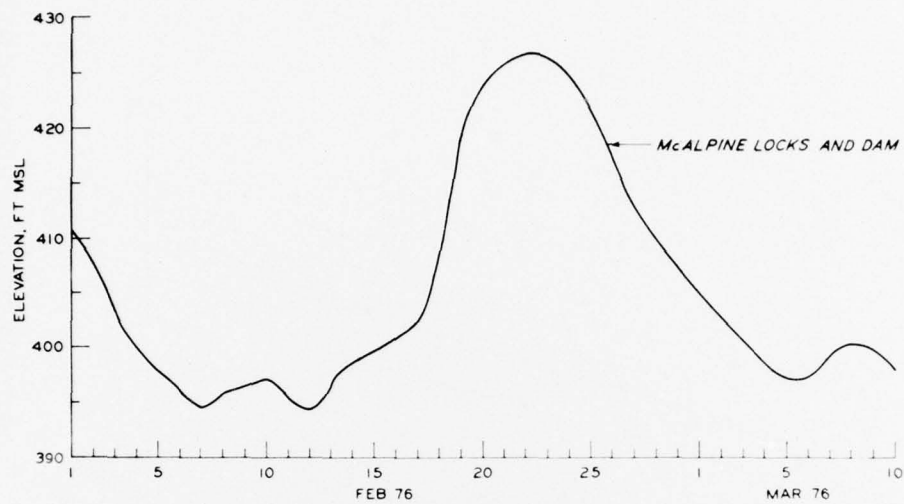
b. SMITHLAND



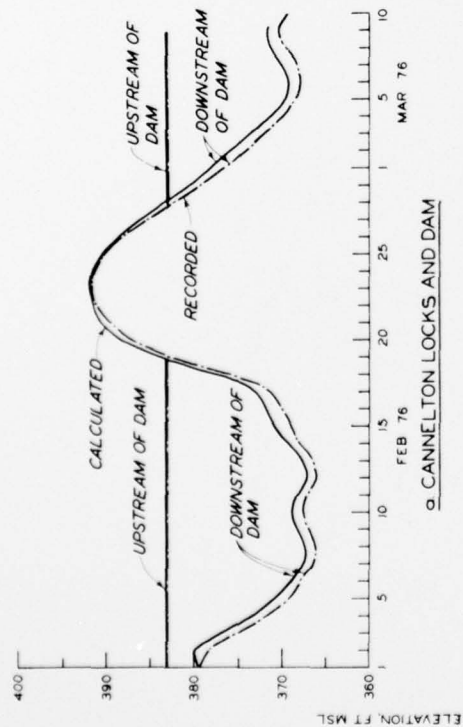
c. CAIRO

1975 ELEVATION HYDROGRAPHS
UNIONTOWN LOCKS AND DAM, SMITHLAND, AND CAIRO

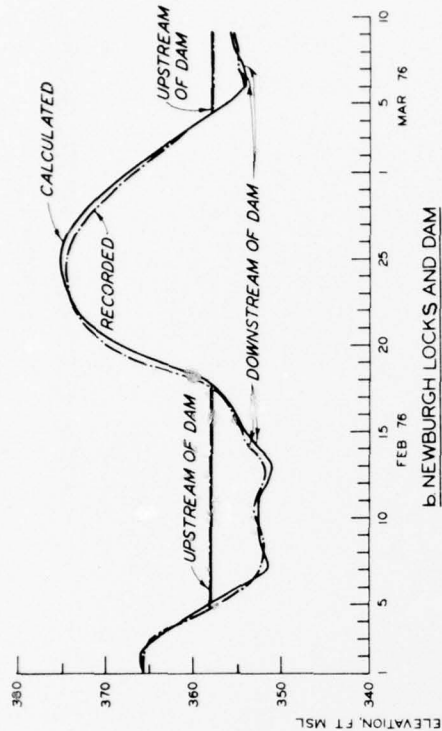




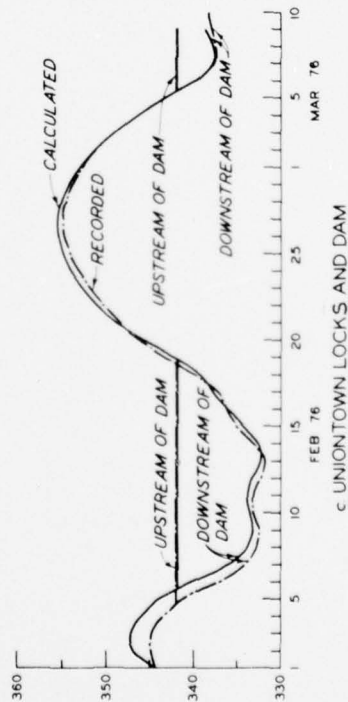
BOUNDARY HYDROGRAPHS
1976 TESTS



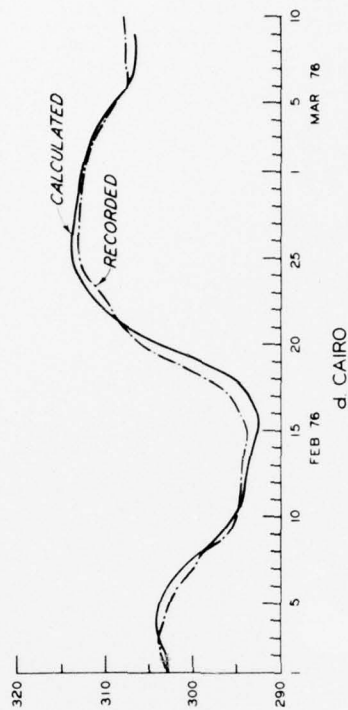
a CANNELTON LOCKS AND DAM



b NEWBURGH LOCKS AND DAM

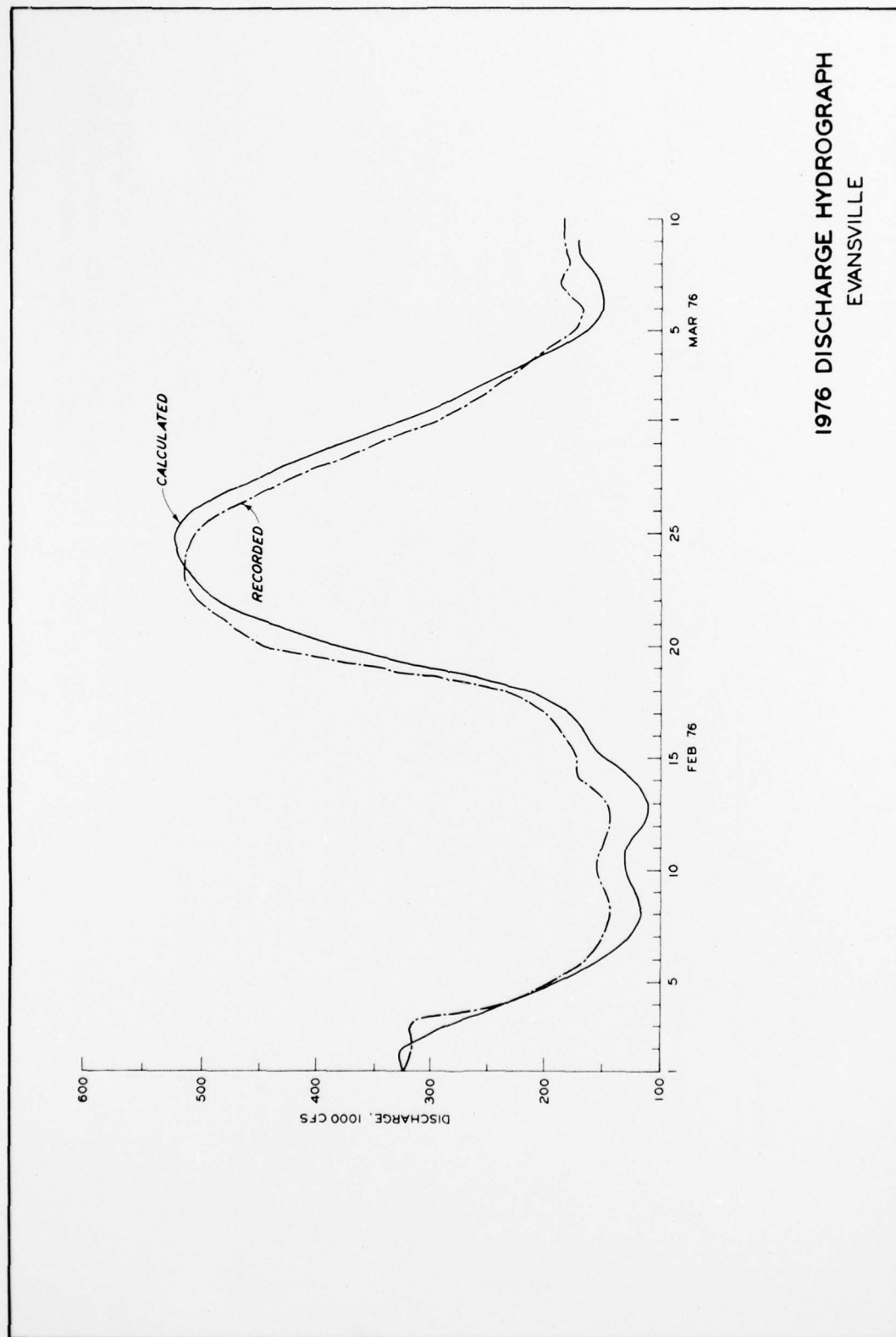


c UNIONTOWN LOCKS AND DAM



d CAIRO

1976 ELEVATION HYDROGRAPHS CANNELTON, NEWBURGH, AND UNIONTOWN LOCKS AND DAMS AND CAIRO



APPENDIX A: LIST OF INPUT REQUIRED BY SOCHMJ

Input is submitted on cards in the following order and format.

1. TITLE (A80)

2. NSTA, NBRCHS, JUNCs, IS5, NSTEP, MNTH, KDAY, KYEAR, LMTE, NØXS
TIME, DT (10I5,2F10.0)

NSTA = Sum of total number of grid points on each branch.
Junction point counts on each branch.

NBRCHS = Total number of branches (max = 19)

JUNCs = Total number of junctions (max = 12)

IS5 = 1 - Elevation, discharge, velocity, area, width,
(hydraulic radius)^{2/3}, Manning's n are printed

= 0 - Elevation, discharge, and velocity are printed

NSTEP = Number of entries in geometric tables (max = 9)

MNTH = Starting month

KDAY = Starting day

KYEAR = Year

LMTE = Print interval

NØXS = Number of gaging stations at which output is desired

TIME = Starting time on 24-hr clock

DT = Time step in seconds

3. STITLE(I), IXS (I,1), IXS (I,2), FF (I,1) (A30,3I5,5X,F10.0)

STITLE(I) = Description of I'th gaging station

IXS(I,1) = Number of the branch upon which the I'th gaging
station is located

ISX(I,2) = The station or net point on a boundary time line
which is immediately upstream of the I'th gaging
station

IXS(I,3) = 1 if gaging station is an upstream boundary or
junction point.

= 0 if gaging station is in the interior of the branch

= -1 if gaging station is a downstream boundary of
junction point

FF(I,1) = The distance in feet from the I'th gaging station to
IXS(I,2).

(One card for each gaging station at which output is desired)

4. NBRCH, IBRNCH (I,1), IBRNCH (I,2), IDIR (I), NEQ (I), ISBR (I),
NFLBR (I,1), NFLBR (I,2), TMILE, DX (I) (8I5,5X,2F10.0)

(One card for each branch)

NBRCH = Number of this branch

IBRNCH (I,1) = Number of first grid point in the I'th branch

IBRNCH (I,2) = Number of last grid point in the I'th branch

If NBRCHS = 2, Set IBRNCH (1,1) = 1 and IBRNCH (1,2) = NSTA and
IBRNCH (2,1) = IBRNCH (2,2) = NSTA.

IDIR (I) = (1) - I'th branch has an upstream outer boundary

= (0) - I'th branch is an interior branch

= (-1) - I'th branch has downstream outer boundary

NEQ(I) = (-1) - Fread's equation relating stage and discharge
is used as the downstream boundary condition of
I'th branch.

= (0) - Elevations are prescribed for boundary
conditions of I'th branch. Always input 0 for a
branch adjacent to and upstream of a dam.

= (1) - Discharges are prescribed for boundary conditions
of I'th branch. Always input 1 for a branch adjacent
to and downstream of a dam.

= (N) - Rating curve is used for boundary conditions
of I'th branch and N values of elevation vs discharge
will be read in.

ISBR(I) = 0 - I'th branch is not a small branch.

= (1) - I'th branch is a small branch.

If ISBR (I) = 0 - leave the next two entries blank.

NFLBR (I,1) = Number of the upstream branch adjacent to the
I'th small branch.

NFLBR (I,2) = Number of the downstream branch adjacent to the
I'th small branch.

TMILE (I) = Extreme upstream mileage of I'th branch.

DX (I) = Spatial step in feet for the I'th branch.

5. DT1, DSI, HSET1, HSET2, HSET3, HSET4, HSET5 (8F10.0)

DT1 = Time step in seconds for small branches. If there are
no small branches DT1 = DT.

DSI = Iteration control for small branches. (0.005 is
probably a good value) If the program is not being used
for the Ohio problem, the remainder of the card is blank.

HSET1 = Upstream elevation which the Cannelton Locks and Dam attempts to maintain.
HSET2 = Upstream elevation which the Newburgh Locks and Dam attempts to maintain.
HSET3 = Upstream elevation on which the Uniontown Locks and Dam attempts to maintain.
HSET4 = Upstream elevation which the Smithland Locks and Dam attempts to maintain.
HSET5 = Upstream elevation which Lock and Dam 52 attempts to maintain.

(If JUNC5 = 0, the next card is omitted)

6. NJUNC, N, (IJUNC (I,J), J = 1,N), (IFLOW (I,J), J = 1,N) (12I5)

(One card for each junction)

NJUNC = Junction number

N Number of branches at this junction

(IJUNC (I,J), J = 1,N) = Branch numbers which comprise the I'th junction

(FLOW (I,J), J = 1,N) = (-1) - J'th branch is downstream of I'th junction

= (1) - J'th branch is upstream of I'th junction

= (2) - Side branch coincides with junction

7. (PRMILE (I), I = 1, NSTA) (38I2)

PRMILE (I) = Print selection control - Input 1's at those net points for which output is desired.

8. (ELEV (K,I), AREA (K,I), R23 (K,I), WIDTH (K,I), CR (K,I), K = 1, NSTEP) (5F10.0)

Geometric data - Begin with first grid or net point on first branch and follow through last grid point on last branch.

9. (ELTAB (I,J), QTAB (I,J), J = 1, NEQ (I)) (8F10.0)

Rating curve for outer boundary of I'th branch if NEQ (I) > 1

10. Q0(I), QP(I), TAU(I), S0(I), H0(I), HP(I), AAVR(I), AK(I) (8F10.0)

There will be one card for each branch with NEQ(I) = -1
Q0(I) = Discharge at beginning of a typical flood in cfs

QP(I) = Peak discharge of a typical flood in cfs
 TAU(I) = Elapsed time from QØ to QP in days
 SØ(I) = Bottom slope of channel in ft/ft
 HØ(I) = Elevation at beginning of a typical flood in ft
 HP(I) = Peak elevation of a typical flood in ft
 AAVER(I) = Cross-sectional area associated with the elevation

$$\left(\frac{HP + HØ}{2} \right)$$

 AK(I) = A value used in computing the kinematic wave velocity.
 Its value ranges from 1.7 to 1.3 with 1.5 a good
 value for most natural channels.

Remember that all the above pertain to the downstream boundary of the branch with NEQ = -1.

11. H(IMIN, 2), Q(IMIN, 2), H(IMAX, 2), Q(IMAX, 2) (5F10.0)

Elevation and discharge at the first and last grid points on the 2nd starting line.

12. H(JJ, 1), Q(JJ, 1), H(JJ + 1, 2), Q(JJ + 1, 2) (8F10.0)

Elevation and discharge at interior grid points of this branch on first and second time lines.

13. QLAST (I), ELAST (I) (5F10.0)

QLAST (I) - Discharge at outer boundary of I'th branch on 3rd time line.

ELAST (I) - Elevation of outer boundary of I'th branch on 3rd time line.

This card is only input if this branch contains an outer boundary.

Cards 11, 12, and 13 (if 13 is applicable) are input in sequence for each branch in the system. If a branch consists of only one grid point such as a branch coinciding with a junction or a downstream boundary, cards 11, 12, and 13 are not input for that branch.

14. NRCH, (IRCH (I), I = 1, NRCH) (12I5)

NRCH - Number of reaches that contain lateral inflow.

IRCH (I) - The upstream station numbers of the reaches that contain lateral inflow.

15. (XINFL (I), I = 1, NRCH) (8F10.0)

XINFL (I) - Third line of lateral inflows

This card is omitted if NRCH = 0

16. IFCNT, (XINFL (I), I = 1, NRCH) (I5, 5X, 7F10.0)/,
(10X, 7F10.0)

IFCNT - Number of time steps until the new lateral inflows which are about to be read apply

XINFL (I) - New line of lateral inflows.

This card is omitted if NRCH = 0.

17. ELAST (II), IECK (II) (F10.0, I5)

ELAST (II) - Elevation specified as the boundary condition for the II'th branch.

IECK (II) - Number of time steps before a new value of ELAST (II) is read in.

If IDIR (II) = 0, this is an interior branch and there will be no card. In any case, this card is present only if NEQ (II) = 0.

18. QLAST (II), IQCK (II) (F10.5, I5)

QLAST (II) - Discharge specified as the boundary condition for the II'th branch.

IQCK (II) - Half the number of time steps before a new value of QLAST (II) is read in.

If IDIR (II) = 0, this is an interior branch and this card is omitted. In any case, this card cannot be present unless NEQ (II) = 1.

Cards 16, 17, and 18 are repeated. Remember that the check on whether to read in new cards or not is first on lateral inflows and then on boundary conditions.

APPENDIX B: NOTATION

A	Cross-sectional flow area
B	Effective width of water surface
g	Acceleration due to gravity
h	Water-surface elevation above mean sea level
h'	River stage
L	Number of net points on large branches
n	Manning's resistance coefficient
N	Average number of iterations required for computations on the small branches within each large time step
q	Lateral inflow per unit distance along channel and per unit time
Q	Discharge
R	Hydraulic radius
S	Number of net points on small branches
S_f	Friction slope (in Table 1)
S_o	Bottom slope of channel
t	Time
V	Mean flow velocity
x	Distance along channel
Δt	Time increment
Δt_l	Time step for large branches
Δt_s	Time step for small branches
Δx	Distance increment
$\partial/\partial t$	Rate of change with respect to time
$\partial/\partial x$	Rate of change with respect to distance

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

Johnson, Billy Harvey

A mathematical model for unsteady-flow computation through the complete spectrum of flows on the lower Ohio River / by Billy H. Johnson. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. : available from National Technical Information Service, 1977.

27, 29, p. ; 8 leaves of plates : ill. ; 27 cm. (Technical report - U. S. Army Engineer Waterways Experiment Station ; H-77-18)

Prepared for U. S. Army Engineer Division, Ohio River, Cincinnati, Ohio.

References: p. 27.

1. Mathematical models. 2. Ohio River. 3. Open channel flow. 4. Unsteady flow. I. United States. Army. Corps of Engineers. Ohio River Division. II. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Technical report ; H-77-18.

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